MODAL ANALYSIS OF A BEAM WITH CLOSELY SPACED MODE SHAPES

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ABSTRACT

This paper investigates the pseudo-root problem by performing modal analysis on a plate specially designed with two closely spaced modes. The first technique used is impact testing and the second technique is sine-sweep shaker testing. Frequency Response Function (FRF) and coherence plots were obtained using LMS Test.Lab. The PolyMAX curvefitting algorithm was used to estimate the natural frequencies. The first twenty modes of the beam were obtained. Two sets of closely spaced modes were obtained at 302Hz and 315Hz and at 546Hz and 551Hz

INTRODUCTION

The Cooper Union offers a sequence of control systems and vibrations classes as part of the required engineering curriculum. Although control systems and vibrations is one of the most weighed topics of study by credit, relatively little research is currently being done by undergraduate students. Use of the lab is mostly due to mini-projects in the ME101: Mechanical Vibration and ME401: Advanced Vibrations courses.

With the acquisition of more and more hardware, the Vibrations Lab will have greater potential for research. In 2010 Professor Baglione (Mechanical Engineering, Vibrations Lab) purchased the SCADAS MOBILE from LMS. This device is a DAQ that is more capable than the current NI DAQs in the Lab. The SCADAS has a frontend with 16 input ports (for impact hammers, accelerometers, transducers, microphones, etc) and 2 output ports for source control. This adds the possibility for multiple-input-multiple-output (MIMO) experiments which is limited with the current DAQs. The source output also facilitates shaker experiments away from the lab since an AC source can be provided by the SCADAS which has a couple of hours of battery life.

This device should be utilized for research at the undergraduate or graduate level because of its capabilities. Efforts should be devoted to learn to use this hardware as well as the Test.Lab software that is required for operation of SCADAS Mobile. One objective is to provide a detailed instructions manual for basic operations of the system (impact testing, shaker testing).
A more academic objective is to use this system to study a plate with closely spaced mode shapes. Professor Karsen from Michigan Technological University has provided the Cooper Union Vibrations lab with such a specially designed polycarbonate plate.

MOTIVATION

This goal is important because finding closely spaced mode shapes is a real problem in the vibrations world, and it is called the pseudo-repeated root problem (Allemang R. J., 2007). One example of this pseudo-repeated root problem was the pogo-effect in the Titan 2 space rocket. In the Titan 2, two of the components had natural frequencies close to that of the resonant frequency of the rocket, causing unwanted longitudinal vibrations along the rocket. Engineers had only measured one natural frequency close to that of the resonant frequency. When they came up with an engineering solution to lower that frequency, they made the vibrations worse because they did not notice or take into account the effects or another frequency. This is a great example of the pseudo-repeated root problem. Since engineers did not initially observe the two frequencies close to the resonant frequency of the structure, their solution took into account only one frequency and therefore did not correct the problem. This was wasted effort, time and money which also could have potentially damaged the rocket (Dotson, 2004).

At the Cooper Union we are studying a simple plate with pseudo-repeated roots to see if we can better understand this experimental issue. This will aid in discovering the limitations of our SCADAS system as well as to improve our experimental modal analysis methods.

BACKGROUND – MODAL ANALYSIS

Modal analysis is the study of the dynamic properties of a structure subject to some input excitation force. It is the experimental method of determining the modal parameters of a structure. This includes the natural frequencies, damping, stiffness, and mode shapes of a system. The assumptions that this theory is based on is that the system is linear and time invariant. The results of modal analysis may be completely invalidated if those assumptions are not true.

EXPERIMENTAL METHODS BACKGROUND

Impact Hammer

As stated previously modal analysis is the study of the dynamic properties of a structure subject to some input excitation force. To obtain FRF's of a structure the structure must be excited at all the frequencies of interest and the output must be measured at all those frequencies. An impact hammer can impart a force on an object as well as measure the force as a function of time (with a load cell at the hammer head). An accelerometer (or many) can be placed on the structure to measure the output of the structure at specific locations. An FRF will be obtained for each impact at a location y and a response at a location x, labeled H_{xy} (i.e. the response at location 1 over the force at location 3 is the FRF H_{13})
Ideally the impact of a hammer acts as an impulse which excites all the frequencies in the frequency domain (impulse in time domain is constant value in frequency domain). This would mean that the impulse of the impact is infinite in magnitude and lasts an infinitesimal amount of time. However this is not realistically possible. Different tips can be screwed on the hammer (plastic, rubber, metal, etc); the harder tips result in impacts closer to impulse functions with an impulse that is higher in magnitude and shorter time.

However, it is not always desired to have such an impact to excite as many frequencies as possible. In a modal analysis experiment, one should excite frequencies of which the response can be measured. If we excite frequencies higher than the bandwidth, as determined by the sampling parameters, then participation of out of band modes distort the data obtained at lower frequencies resulting in poor coherence. For this reason, different tips are used depending on what frequencies you want to excite.

Ideally when you strike a structure you want to impact it only once to get clean data. However, because the impact can occur so quickly that the structure may vibrate fast enough to hit the hammer again before the user pulls it away. This results in a "double-impact" (or more). This is a skill that is obtained over time and care should be taken to try to reduce double-impacts when possible. This is sometimes unavoidable, and averaging of many samples helps reduce the error.

Shaker

Impact testing acts to excite all the natural frequencies of a system but higher frequencies are attenuated. Which frequencies are attenuated depends on the choice of the tip used. This is not a user-friendly system and requires much care to get clean data.

A shaker however does away with all of this by exciting a system at specific frequencies of interest. A stinger that is attached to shaker and is screwed into the test piece forces the vibration. The system can be excited by different functions, such as a sine curve swept from a lower frequency to higher frequency or a burst random signal with a specific frequency range. In these experiments a sine curve was swept from 0Hz to 2048Hz (in one measurement period). In LMS such a function is called a Periodic Chirp and is shown in Figure 1 in the time domain.

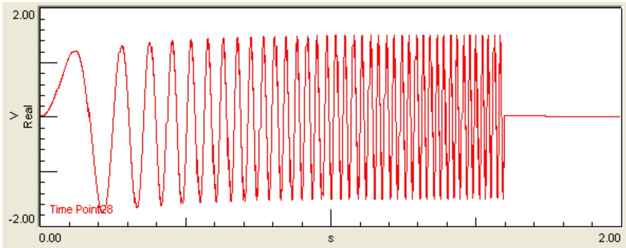

Figure 1 LMS Periodic Sine Chirp Curve

EXPERIMENTAL

Plate

The polycarbonate plate is 53 cm by 32 cm. Twenty points were marked on the plate in a 4X5 grid for each of the test locations. However, for shaker testing another point was added since a pre-drilled hole existed there for the shaker stinger to be attached. Therefore 21 test points were used for shaker testing. For the impact testing, the accelerometer was placed at Point 7 and each of the first 20 points were impacted. This is called a roving hammer test since the hammer moves with an accelerometer in place. For the shaker testing, the shaker was attached at Point 21 while the accelerometer was moved to each of the 21 points.

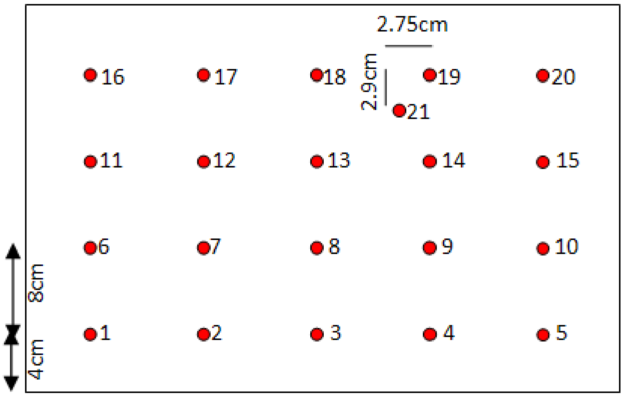

Figure 2 Plate Measurement Point Diagram

LMS

The LMS SCADAS is turned and plugged into an outlet (although it can run on battery if needed). Special cables are used to connect BNC cables into the frontend of the SCADAS. The accelerometer is attached to the second input in the frontend of the SCADAS. The reference which is either the impact hammer or the force transducer in a shaker test is plugged into the first input. A USB cable connects the SCADAS to a computer with licensed Test.Lab software.

Test Setup

The plate is suspended on two rubber tubes which are tied across two vertical fixtures on the optical table in Acoustics Laboratory at the Cooper Union. This aims to constrain the plate as little as possible to simulate a free-free setup. Note that the rubber tubes should both be as equally taut as possible. Figure 3 shows the setup.

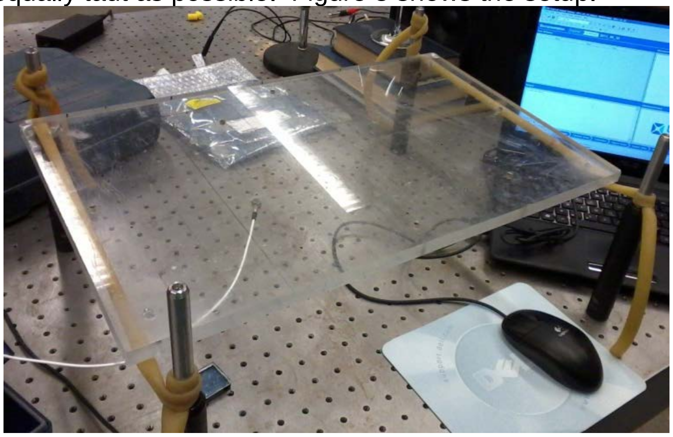
Figure 3 Plate Boundary Conditions

Impact Hammer

A model 086C03 PCB Piezotronics Impact Hammer was used to impact the beam. A hard plastic tip was used because it had the best impacts. The force data obtained when using a hard plastic tip was much cleaner than with the other tips used.

A model A352C65 PCB Piezotronics Accelerometer was used to measure acceleration. The accelerometer was placed at Point 7 and the beam was impacted at Points 1 to 20. Five averages were taken at each point. Figure 3 shows the impact hammer setup.

Shaker

A model K2007E01 Modal Shop Shaker and model 208C01 PCB Piezotronics Force Transducer were used for shaker testing. A similar setup was used for shaker testing as for impact hammer testing. The stinger was attached at Point 21.

A model A352C65 PCB Piezotronics Accelerometer was used to measure acceleration. The accelerometer was placed at Points 1 to 21. Ten averages were taken at each point. Figure 4 shows a picture of the shaker-setup.

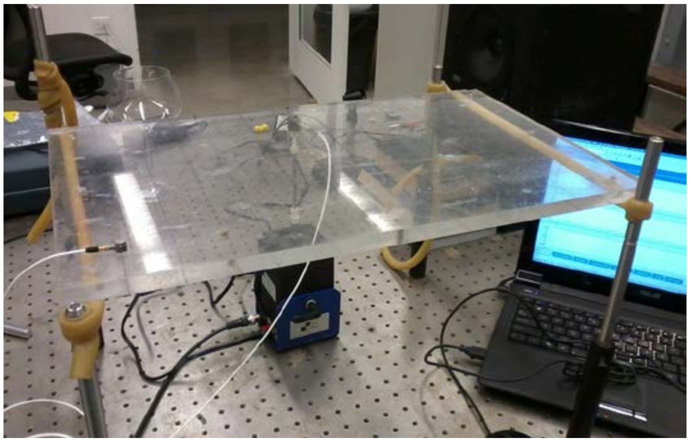
Figure 4 Shaker Testing Setup

RESULTS AND DISCUSSION

Impact Hammer Results

The FRF magnitude and phase are shown in Figure 5 below for the drive point. The coherence is also displayed on the magnitude graph.

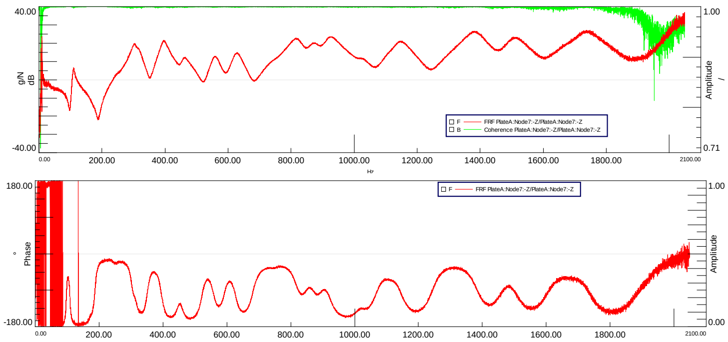

Figure 5 FRF Magnitude (top) and Phase (bottom) For Drive Point Measurement in Impact Test

Our magnitude plot indicates that our system is highly damped since our natural frequencies do not exhibit sharp peaks and neither do the anti-resonances. Our magnitude shifts by -180 degrees and by 180 degrees at anti-resonances, as expected for a drive point measurement. The coherence is very high which means that there is little noise or leakage in the system, except for higher frequencies greater than 1800Hz. This is expected because our reference function attenuates at higher frequencies.

In Figure 6 we can see the coherence zoomed in between 0.99 to 1. We see that for the most part the coherence is higher at anti-resonances than at the nearby resonances. This is not the case for the first mode shape at around 108Hz. The anti-resonance has a much lower coherence than the resonance in that region. This may be due to leakage. However there is no indication of the source of this leakage.

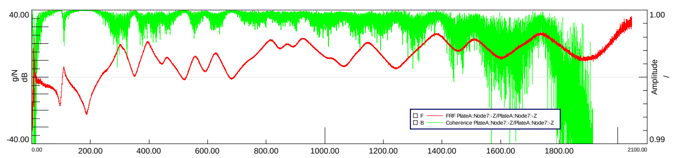

Figure 6 Coherence

Figure 7 shows the real and imaginary components of the FRF. We see that the imaginary component reaches relative minima at the same frequencies that the real component crosses the x-axis. This again is expected for a drive point measurement.

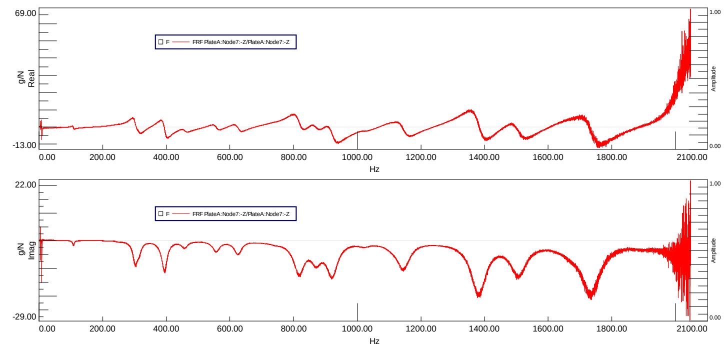

Figure 7 FRF Real (top) and Imaginary (bottom) For Drive Point Measurement In Impact Test

Table 1 Natural Frequencies and Damping for Impact Testing

F	Damping
108	3.07%
250	3.60%
302	2.72%
315	2.65%
394	2.68%
458	2.56%
557	2.35%
626	2.21%
745	2.39%
818	2.47%
870	2.25%
922	2.25%
1020	1.54%
1108	1.95%
1145	2.01%
1381	2.10%
1506	2.04%
1642	1.35%
1730	1.94%
1887	0.47%

The stabilization diagram can be seen in Figure 8 along with the selected poles. The red curve is the sum of all the FRF's and the green curve is the FRF for the drive point measurement. The values for the natural frequencies and damping ratios are tabulated in Table 1.

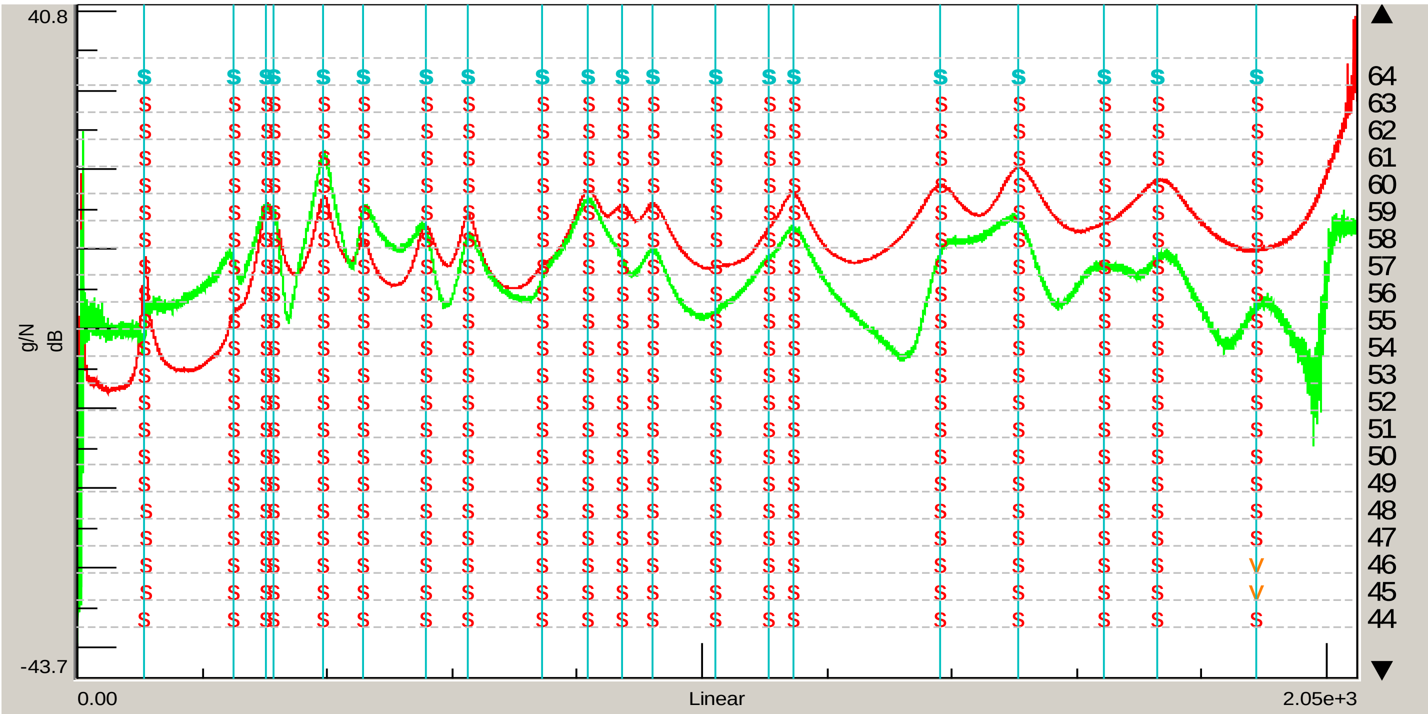

Figure 8 Stabilization Diagram and Selected Poles for Impact Test

Shaker Results

Figure 9 below shows the FRF function magnitude and phase. The coherence is also shown on the magnitude graph.

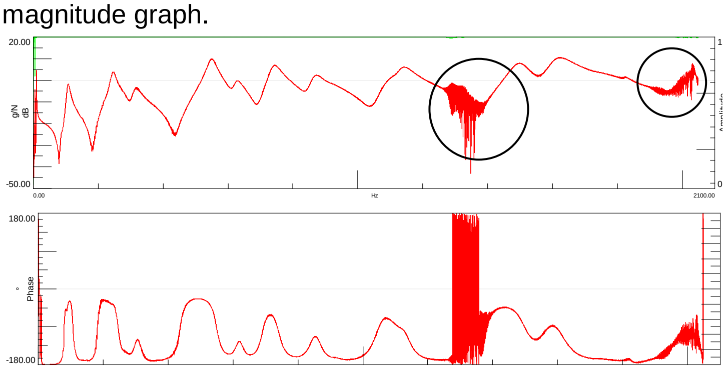

Figure 9 FRF Magnitude (top) and Phase (bottom) For Drive Point Measurement In Shaker Test

We observe the same properties as before in the phase shift. However, the shaker test gives much cleaner data. There seems to be less noise for both magnitude and phase. This is also manifest in the coherence function which is much higher than for the impact test. The only exceptions are the two regions circled above. However if we look at the force spectrum in Figure 10 we see that the output of the shaker is noisy at these exact same regions. One possibility is that these may frequencies may excite the natural frequencies of the stinger. It is also possible that something else on the table was being excited at these frequencies which would cause unwanted vibrations on the table that could have made its way to the plate. During my last few runs I noticed my laptop making a rattling noise, coming from within, at higher frequencies. This was not observed until the very last runs and so the laptop was not moved so that the data would all be consistent.

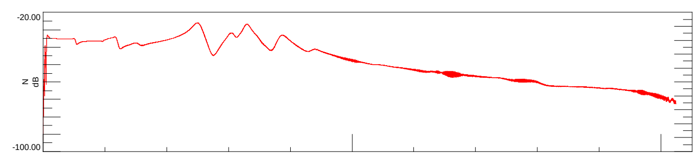

Figure 10 Force Spectrum For Drive Point Measurement in Shaker Testing

Figure 11 shows a closer look at coherence from 0.99999 to 1. The coherence is relatively low at the two regions discussed above, however its value is still very high for such a messy portion on an FRF. If there is some unmeasured input to the system as discussed above then the coherence should be much lower. Coherence should be further analyzed.

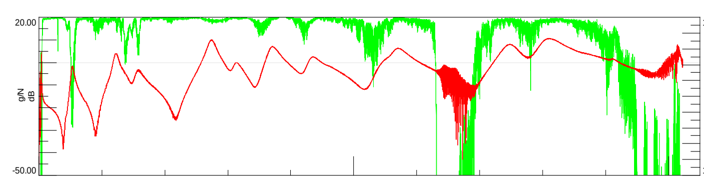

Figure 11 Coherence

Figure 12 shows the real and imaginary components of the FRF. This data although it looks less noisy than the impact data, it doesn't show the same behaviors of a drive point that the impact data does. The real component is not always zero where there are minima in the imaginary component.

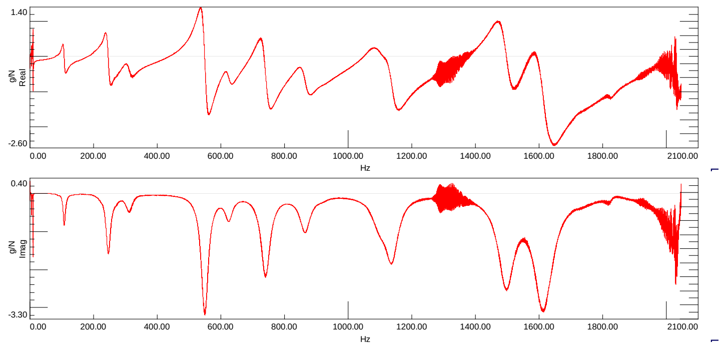

Figure 12 FRF Real (top) and Imaginary (bottom) For Drive Point Measurement in Shaker Test

Table 2 Natural Frequencies and Damping for Shaker Testing

F	Damping
108	3.97%
246	3.64%
312	3.22%
396	2.58%
457	2.27%
546	1.36%
551	2.38%
625	2.23%
741	2.34%
816	1.99%
866	2.25%
920	2.20%
1101	2.58%
1138	2.01%
1498	2.01%
1613	2.06%
1734	1.55%

The stabilization diagram can be seen below in Figure 13**.** along with the selected poles. The red curve is the sum of all the FRF's and the green curve is the FRF for the drive point measurement. The values for the natural frequencies and damping ratios are tabulated in Table 2.

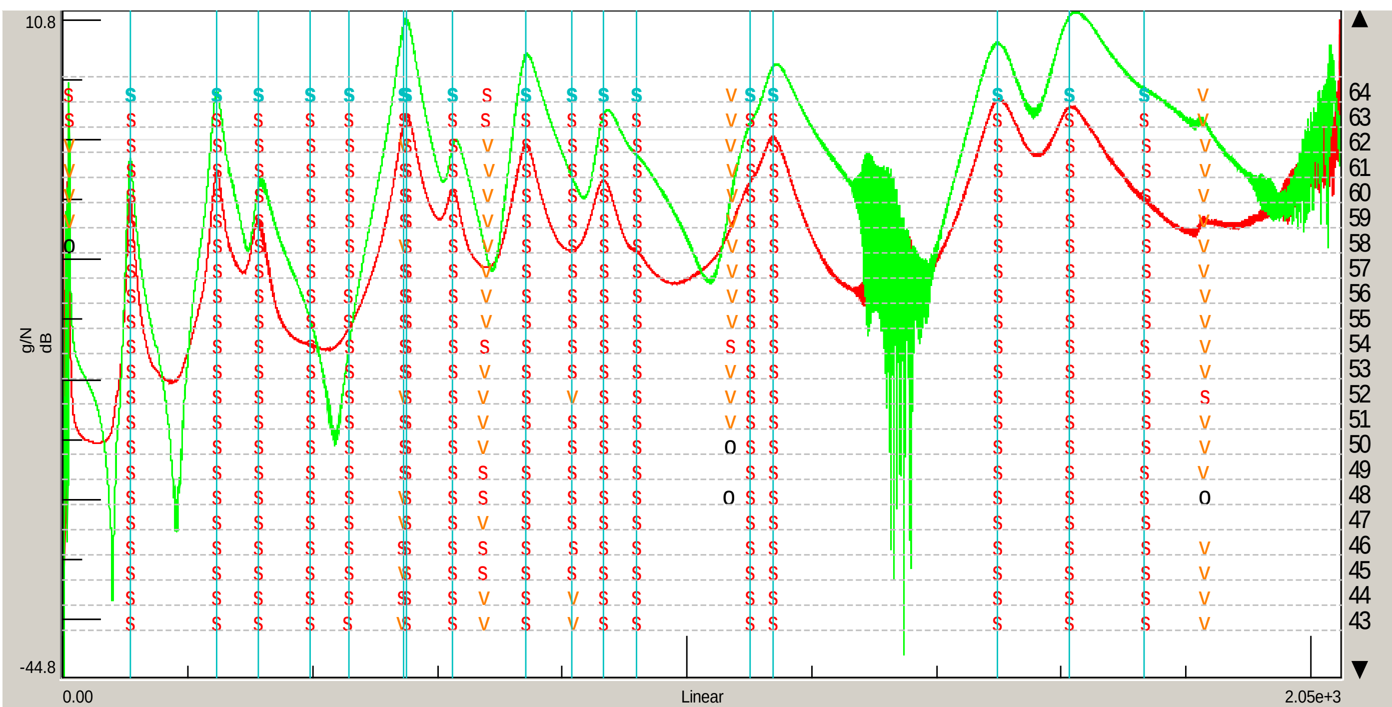

Figure 13 Stabilization Diagram and Selected Poles for Shaker Test

The natural frequencies of both methods are tabulated in Table 3 for comparison.

Table 3 Natural Frequencies Obtained From Both Methods

Impact	108	250	302	315	394	458		557	626	745	818	870	922	1020	1108	1145	1381	1506	1642	1730	1887
Shaker	108	246		312	396	457	546	551	625	741	816	866	920		1101	1138		1498	1613	1734	

The two methods give very similar results for the most part. However there are four modes that are not detected by both methods. Furthermore we see two sets of frequencies that are very close to each other. Each method detects only one of the frequencies in these two pairs that have been highlighted in blue.

CONCLUSIONS AND FUTURE WORKS

Two methods have been used to find the natural frequencies of a plate. The methods agree for the most part, however they do not both give exactly the same results. Each method misses some of the natural frequencies. Two pairs of closely spaced mode shapes have been identified at 302Hz and 315Hz and at 546Hz and 551Hz.

The data presented here is much cleaner than data acquired with the test-rig used previously in Vibrations Lab. Therefore, use of the optical table, in the Acoustics Lab is recommended. It is also recommended that the optical table be cleared of other objects when running shaker experiments, because the shaker might excite other objects which can cause vibrations on the surface of the table that would be only lightly damped by the metal fixtures.

The test set up can further be improved by using new rubber tubes of the same length, which would ensure that the rubber tubes holding the plate are have the same tension. Results may also improve if the shaker is placed on some rubber mat on the surface of the optical table so that shaker vibrations would not be transmitted to the table and up along the optical fixtures.

Further studies can be done by varying the bandwidth. The current bandwidth 0-2048 Hz may be too high to provide much detail on closely spaced mode shapes. Shaker testing on a smaller frequency range may reveal more information in the frequency domain.

In addition, the coherence should be analyzed. Although a high coherence implies that our data is good, the data is suspect since high coherence results at places with significant noise in the FRF and at anti-resonances.

Lastly, the curve-fitting functions in Test.Lab are described in the help files. An understanding of these curve-fitting techniques should be achieved to truly be confident in the data obtained. Research should be done on curve-fitting techniques and on the specific ones used by Test.Lab to fully analyze this data.

ACKNOWLEDGMENTS

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REFERENCES

Allemang, R. J. (2007). *Vibrations: Experimental Modal Analysis.* Univ of Cincinnati.

Allemang, R. J. (2008). *Vibrations:Analytical and Experimental Modal Analysis.* Univ of Cincinnati.

Baglione, M. (2011). ME401:Advanced Vibrations Class Notes. NY: The Cooper Union.

Dotson, K. (2004, Winter). Mitigating Pogo on Liquid-Fueled Rockets. *Crosslink* .

Inman, D. J. (2008). *Engineering Vibration: 3rd Edition.* New Jersey: Pearson Education, Inc.

Modal Analysis. (n.d.). Retrieved 2011, from LMS Site: http://www.lmsintl.com/modal-analysis

VanKarsen, C. *MEEM470 Course Notes.* Michigan Tech, 2008.