The Cooper Union
Department of Electrical Engineering
ECE135 Engineering Electromagnetics
Exam III
May 7, 2012

Time: 3 hours. **Closed book, closed notes. Calculator provided.**

Part I: Maxwell's Equations and Constitutive Relations. 20pts. total.

1. State Maxwell's equations in **differential** form for the following cases:
   (a) In the time-domain, in the most general case.
   (b) In the phasor domain, assuming a linear homogenous isotropic lossy medium (σ, μ, ϵ), but $\rho = 0$.

2. Write the general relation among $\vec{D}, \vec{E}, \vec{P}$ and give units for all quantities that appear.

3. Write the general relation among $\vec{B}, \vec{H}, \vec{M}$ and give units for all quantities that appear.

Part II: Answer all problems.

1. **[2 pts.]** Short answers:

   (a) An infiinitesimal dipole antenna is called a _________ dipole.

   (b) True or false: The current distribution on a straight-wire antenna is tapered to 0 at the endpoints.

2. **[4 p.ts]** When $2A$ is applied to a lossless antenna, $100W$ of power is radiated. You have enough information to determine an antenna parameter. Name it, and compute its value.

3. **[6 pts.]** Consider an antenna with radiation pattern $K(\theta, \phi)$, and total radiated power W.

   (a) What are the units of K?

   (b) Write a formula for the power density at a point (r, θ, ϕ), in W/m^2.

   (c) Write a formula for the directive gain $g_D(\theta, \phi)$.

   (d) Suppose this antenna is used in an array, with an array pattern with directivity $20dB$. If you know g_D for the elemental antenna, do you have information to determine the directivity of the array? If yes, what is it? If not, what more information would you need?

4. **[6 pts.]** The total field of the magnetic dipole antenna (loop of radius a centered at the origin in the xy-plane) is, in spherical coordinates:

$$
\begin{aligned}
E_\phi &= -\frac{j\omega\mu I a^2}{4} e^{-jkr} \left(\frac{jk}{r} + \frac{1}{r^2} \right) \sin\theta \\
H_r &= \frac{j\omega\mu I a^2}{4} e^{-jkr} \left(\frac{2}{\eta r^2} + \frac{2}{j\omega\mu r^3} \right) \cos\theta \\
H_\theta &= \frac{j\omega\mu I a^2}{4} e^{-jkr} \left(\frac{j\omega\varepsilon}{r} + \frac{1}{j\omega\mu r^3} + \frac{1}{\eta r^2} \right) \sin\theta
\end{aligned}
$$

   and the other field components are zero.

   (a) Write the terms for the near-field approximation. Examine the near-field to determine: whether the power density is purely real, purely reactive, or a mix; the dependence of power with distance (i.e.,. $1/r^m$: what is m?); whether there is a component of the (complex) Poynting vector that is NOT radial. *Do not compute the actual Poynting vector; do not invoke results you remember- show how your near-field approximation leads to the results.*

   (b) Write the terms for the far-field approximation. Then repeat part a above to describe the (complex) Poynting vector without actually computing it.

5. **[2 pts.]** Consider an aperture antenna modeled with equivalent sources $\vec{J}_s$ (surface electric current) and $\vec{M}_s$ (surface magnetic current). Pick the one that is NOT TRUE!! (One is false, the others are true).

   (a) $\vec{J}_s$ depends on $\vec{H}$ tangential to the aperture;

   (b) $\vec{M}_s$ depends on $\vec{E}$ tangential to the aperture;

   (c) if we are interested in the far-field, there is no need to model the normal $\vec{E}$ and $\vec{H}$ components with equivalent sources;

   (d) the total $\vec{E}, \vec{H}$ fields in the far-field are represented by the single vector potential $\vec{A}$.

6. **[7 pts.]** This question concerns guided waves in cylindrical waveguides. Short answer:

   (a) Briefly define (one sentence or less): dominant mode.

   (b) Briefly define (one sentence or less); degenerate mode.

   (c) A single mode propagating in an "ideal" material can nevertheless experience dispersion. This type of dispersion is called __________ dispersion, and can be mitigated by operating at *(higher/lower)* frequencies.

   (d) The wave impedance for a mode will be purely reactive if operated *(above/below)* the cutoff frequency. This results in a(n) ____________ wave.

   (e) True or False (no justification): When establishing the scattering parameter model, the "voltage" is computed as the integral of the $\vec{E}$-field in the cross section, and the "current" is computed as the integral of the surface current in the cross-section.

7. **[4 pts.]** For a TE or TM mode in a cylindrical closed boundary waveguide: Draw a sketch of the "universal curve" for ω versus β, and use this curve (NOT FORMULAS) to show that the group velocity *(increases/decreases)* (which?) as you increase frequency.

8. **[4 pts.]** Give the two main reasons many practical devices are realized by employing the TE_{10} mode in a rectangular waveguide. Such devices will be limited to operating within a certain frequency range; give the upper and lower frequencies (I don't mean a formula; I mean identify these frequencies and WHY we are limited to operate between them: be BRIEF but clear).

9. **[9 pts.]** Given a $3cm \times 5cm$ rectangular waveguide filled with a dielectric with $\epsilon_r = 9$.

   (a) Identify the TM mode with lowest cutoff frequency (i.e., specify the mn values for this TM_{mn} mode).

   (b) For this mode, find the cutoff frequency.

   (c) For a frequency 1.5 times cutoff, find the guide wavelength.

   (d) List all other modes (if any) with a cutoff frequency the same or lower than this mode. Do not actually compute these cutoff frequencies if you don't have to!!

10. **[4 pts.]** Consider a cavity resonator with a conducting boundary. Assume the conducting walls are characterized by surface resistance R_s, and the dielectric (ϵ, μ) is low-loss with conductivity σ.

   (a) Give the EXACT definition for Q (in general).

   (b) Write an explicit formula for Q. This formula should involve integrals of field quantities $\vec{E}, \vec{H}$ (No other field quantities should appear!!). Use $\int_S dS$ to denote integration over the conducting boundary, and $\int_V dV$ to denote integration over the volume of the resonator. When doing the integral on the surface, you must show which components (i.e., tangential and/or normal) are involved.

   (c) You can use your formula for Q to obtain a formula for the $3dB$ bandwidth of the resonance; this formula should involve the integrals only, and should not (explicitly) involve the resonant frequency! Obtain this formula.

   (d) Even if you could not finish b,c, you can still answer this: is the formula for the $3dB$ bandwidth in part c and exact or approximate formula?

11. **[5 pts.]** Given the following scattering matrix of a 2-port:

$$S = \begin{bmatrix} 0.8 & -j0.4 \\ j0.4 & 0.9 \end{bmatrix}$$

   (a) Compute the power dissipation matrix.

   (b) Is the network lossless?

   (c) Is the network reciprocal?

12. **[4 pts.]** A scattering matrix S is defined for an N-port relative to a reference impedance Z_0. If a_i, b_i denote the incident and reflected waves at port i, and V_i, I_i the voltage and current at port i:

   (a) Write the general relation for a_i, b_i in terms of V_i, I_i.

   (b) If port i is terminated in a load $Z_L = Z_0$, then **use your formulas** (not just something you may have memorized) to specify the condition imposed on a_i and/or b_i.

   (c) In general, the power flowing in at port i is $\frac{1}{2}\,\mathrm{Re}\,(V_i I_i^*)$. Write this in terms of a_i, b_i instead.

13. **[3 pts.]** Let P_{diss} be the power dissipation matrix of an N-port. Specify the conditions on the eigenvalues for each of the following:

   (a) passive

   (b) lossless

   (c) active

14. **[6 pts.]** Consider a lossless circulator. Assume "symmetry" in the sense that the return loss, isolation, etc. are the same regardless of which port is used as the input. Given that $|S_{12}| = 0.9$, and the return loss is $20dB$.

    (a) Draw a symbol for the circulator, with the ports numbered.

    (b) Compute the magnitude of every scattering parameter (on a linear scale).

    (c) Compute the insertion loss, isolation and directivity, all in dB.

15. **[6 pts.]** Consider a directional coupler Assume "symmetry," for example the primary and secondary lines are actually interchangeable. Call $1-2$ the primary line, and $3-4$ the secondary line, with $1 \to 2$ and $3 \to 4$ considered "forward" directions.

    (a) Each of the four quantities $|S_{i2}|$, $1 \leq i \leq 4$, expressed in a dB scale, has a "name." For example, one of them is called the (through) insertion loss. Give the names, and identify each as $|S_{i2}|$

    (b) Using two of these four names, we can introduce a new quantity as _____ = _____ - _____ . Write this (in words).

16. **[8 pts.]** Refer to Figure 16. A three-port network, with ports labeled $1, 2, 3$, respectively, is comprised of a tee (ports labeled $1A, 2A, 3A$) with scattering matrix:

$$S_A = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \end{bmatrix}$$

and a directional coupler (ports labeled $1B, 2B, 3B, 4B$) with:

$$S_B = \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} & 0 & j\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & -j\frac{1}{\sqrt{2}} & 0 \\ 0 & -j\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ j\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \end{bmatrix}$$

Port $3A$ of the tee is connected to port $1B$ of the coupler through a waveguide segment with $\lambda/4$ length. Port $2A$ of the tee is terminated in a short circuit, and port $4B$ of the coupler is terminated in a matched load. The resulting 3-port is determined by the associations $1 = 1A$, $2 = 2B$, $3 = 3B$.

GIVEN: (You do not have to check the following, but it is true): Both the tee and coupler are lossless.

DO NOT COMPUTE THE WHOLE SCATTERING MATRIX OF THE THREE PORT!!!! ONLY DO THE FOLLOWING!!!

(a) By inspection (brief justification) determine if the overall three-port is: lossless, passive, reciprocal.

(b) Compute the scattering parameters (only): S_{11}, S_{12} and S_{32}. Compute these parameters directly, i.e., do not use reciprocity or other "tricks." Give brief explanation of what you are doing, i.e., don't just write down a final answer; even if correct, I may mark you wrong if I don't see your work (e.g., if something turns out to be 0, you may have the right answer even if your method is wrong).