The Cooper Union
Department of Electrical Engineering
ECE135 Engineering Electromagnetics
Exam II
April 12, 2012

Time: 2 hours. **Closed book, closed notes. Calculator provided.**

For oblique incidence of plane waves: $Z_{TE} = \eta/\cos\theta$, $Z_{TM} = \eta\cos\theta$

Part I: Maxwell's Equations and Constitutive Relations. 20pts. total.

1. State Maxwell's equations in **differential** form for the following cases:
   (a) In the time-domain, in the most general case.
   (b) In the phasor domain, assuming a linear homogenous isotropic lossy medium (σ, μ, ϵ), but $\rho = 0$.

2. This question deals with material properties.
   (a) Define an electric dipole moment vector $\vec{p}$ (draw a sketch), and use it to "derive" the appropriate units for $\vec{P}$.
   (b) Define a magnetic dipole moment vector $\vec{m}$ (draw a sketch), and use it to "derive" the appropriate units for $\vec{M}$.
   (c) $\epsilon_r - 1$ is a special quantity that has a symbol and a name– write the standard symbol and name it.
   (d) $\mu_r - 1$ is a special quantity that has a symbol and a name– write the standard symbol and name it.

Part II: Answer all problems.

1. **[8 pts.]** Refer to Figure 1: the region between two perfectly conducting coaxial cylinders, of length ℓ and radii a, b, respectively, is partially filled with air, and partially filled with a dielectric material (ϵ) that fills 1/3 of the interior. A total charge Q is on the inner conductor, and a total charge $-Q$ is on the outer conductor.

   (a) Determine $\vec{D}$ and $\vec{E}$ in the region $a < r < b$, in each material.

   (b) Compute the capacitance, C.

2. **[8 pts.]** Refer to Figure 2. $N = 50$ turns of wire carrying current I are wound tightly around an air core and a permanent magnet with $\vec{M}_0 = 10\hat{a}_x$ (with standard units as you wrote in part I of this test). The air region and permanent magnet each have length $2cm$. The cross-sectional area is $A = 0.1cm^2$. Neglect fringing effects. **Note:** You can leave your answers in terms of μ_0, but otherwise compute all constants that appear.

   (a) Find $\vec{B}$ and $\vec{H}$ (as a function if I) in the air, and in the permanent magnet.

   (b) Find the inductance, L.

   (c) Is it possible for I to be such that $\vec{B} = \vec{0}$ in the permanent magnet? If so, find the value. Note that $I > 0$ in the direction shown (if you need the reverse direction, report $I < 0$).

   (d) Is it possible for I to be such that $\vec{H} = \vec{0}$ in the permanent magnet? If so, find the value. (Follow the sign convention for I as noted above).

3. **[5 pts.]** Refer to Figure 3. A non-uniform surface charge density $\rho_s(x) = x$ is distributed over the region $\{-a/2 \le x \le a/2, \ -b/2 \le y \le b/2\}$ in the xy-plane. Assume free-space.

   (a) Set up explicit integral expressions for $\vec{E}(0,0,z)$ and $V(0,0,z)$ for points along the z-axis.

   (b) Examine the integrals (do not invoke symmetry arguments from the physical setup itself) to determine whether E_x, E_y, E_z and V are > 0, $= 0$ or < 0 for points on the z-axis.

4. **[3 pts.]** Write the general integral formula for $\vec{A}$ in terms of $\vec{J}$ (assume STATIC fields in free-space), IN RECTANGULAR COORDINATES. That is, x, y, z, etc. and not $\vec{r}$, etc. should appear in the integral.

5. **[6 pts.]** A steady current I flows in a wire shaped like a square in the xy-plane, centered at the origin, with side length a, as shown in Figure 5. Assume free-space. This problem concerns the $\vec{A}$ field. Do NOT attempt to set up an explicit integral formula for this specific case.

   (a) We can immediately say either $A_x = 0$, $A_y = 0$ or $A_z = 0$ everywhere. Which?

   (b) What is the direction of $\vec{A}(x, 0, z)$ for $x > a/2$? That is, points in the xz-plane ($y = 0$) to the right of the square loop. In particular, specify whether each component is > 0, $= 0$ or < 0. *Do this by invoking symmetry arguments: draw a sketch showing the "influence" of current at different points on the loop.*

6. **[6 pts.]** The phasor electric field for a plane wave in a lossless dielectric ($\epsilon_r = 9$, $\mu_r = 1$) is given by:

$$\vec{E} = (E_x \hat{a}_x + E_y \hat{a}_y + 4\hat{a}_z)\, e^{-j(4y+3z)} \qquad [V/m]$$

   where E_x, E_y are possibly complex constants.

   (a) Find $\vec{k}$ (vector), λ (in the material, not free-space), v, η, and the frequency f in Hertz.

   (b) Specify conditions on E_x, E_y for this to be a valid plane wave.

   (c) Specify E_x, E_y for which this is RIGHT circularly polarized, and LEFT circularly polarized. You must justify that you have these correct, i.e., right vs. left. (You do not need to go back to the time-domain for this; use a rule we discussed in class).

7. **[6 pts.]** Refer to the SAME FIELD as in Problem 6, which is now assumed to be RIGHT CIRCULARLY POLARIZED. It is incident on a perfect conductor at the $z = 0$ plane. Hypothesize a reflected plane wave given by:

$$\vec{E}_r = (E_{xr} \hat{a}_x + E_{yr} \hat{a}_y + E_{zr} \hat{a}_z)\, e^{-j(4y-3z)} \qquad [V/m]$$

   NOTE THE CHANGE IN THE EXPONENTIAL FACTOR FROM THE INCIDENT WAVE!!

   (a) Is this oblique incidence or normal incidence and, if oblique, is the plane of incidence: xy-plane, xz-plane, yz-plane?

   (b) Impose the condition that the reflect field is a valid plane wave, and also impose appropriate boundary conditions for the total $\vec{E}$ field (don't worry about the boundary conditions for $\vec{H}$) to determine E_{xr}, E_{yr}, E_{zr} (yes, all three values can be found uniquely!).

   (c) Is the reflected wave circularly polarized? DON'T CHECK if it is left or right circularly polarized, just whether it is circularly polarized at all!! Don't simply invoke a result you may remember from class; show that the reflected wave either does or does not have the form of a circularly polarized wave.

8. **[5 pts.]** A lossy material (σ, ϵ, μ) has $\alpha = 0.2\beta$ (α in Np/m and β in rad/m) at a certain frequency.

   (a) Find the value of the loss tangent. **Note:** Yes you have enough information to compute it!

   (b) Would this material be better characterized as a low-loss dielectric or a good conductor? Even if the loss tangent value is not "extreme" enough to make the approximations discussed in class very accurate, you can still classify the material as more one than the other. **Note:** Even if you couldn't solve part a, at least indicate what "test" you would apply here.

9. **[5 pts.]** Consider oblique incidence of plane waves (lossless case). Short answer– no justification needed.

   (a) Under condition of TIR, what can be said about the reflection coefficient? Pick one: $|\Gamma| = 1$, $\Gamma = 0$ or Γ real.

   (b) Does the critical angle depend on TE vs. TM polarization?

   (c) For incidence at the Brewster angle, what can be said about the reflection coefficient? Pick one: $|\Gamma| = 1$, $\Gamma = 0$ or Γ real.

   (d) Does the Brewster angle depend on TE vs. TM polarization?

   (e) Incidence at the Brewster angle, or incidence above the critical angle– which causes the transmitted wave to be evanescent?

10. **[3 pts.]** Consider a boundary between glass with $\epsilon_r = 4$ and air.

    (a) Can TIR occur going from air onto glass, glass onto air, both or neither? No justification needed.

    (b) Compute the critical angle, θ_c.

11. **[5 pts.]** Refer to Figure 11. A dielectric coating of thickness $0.3cm$, with $\epsilon_r = 2$, is placed on the surface of a dielectric substrate with $\epsilon_r = 5$. A y-polarized plane wave with $\lambda_0 = 0.2cm$ in air is incident on the dielectric coating at an angle of $30°$.

    Draw an equivalent transmission line model. Label each segment with the appropriate computed characteristic impedance, and specify the value of ℓ/λ (length divided by wavelength [in the material, not free-space]) for the coating layer. **Do not compute more than I'm asking for; e.g., do not compute the reflection coefficient.**

12. **[12 pts.]** A 100Ω transmission line is terminated in a $50+j70\Omega$ load. Use a Smith chart to design a series s.c. stub tuner with characteristic impedance 75Ω. In particular:

- Sketch the transmission line setup.
- Use a calculator AS LITTLE AS POSSIBLE!!
- Be sure to give both solutions, for the length of the stub and distance from the load.

13. **[8 pts.]** A 50Ω transmission is terminated in an unknown load. Voltage minima are observed at $4cm$ and $10cm$ from the load. The maximum voltage amplitude on the line is $3V$, and the minimum voltage amplitude is $2V$.

    (a) Determine the locations of the two voltage maxima nearest the load. Sketch the standing-wave diagram (for a section long enough to show two maxima and two minima).

    (b) What is the VSWR? (Linear scale, not decibels)

    (c) Use a Smith chart to determine: the value of the load impedance; the value of the equivalent impedance seen at a voltage maximum; the value of the equivalent impedance seen at a voltage minimum. (Give all impedances in Ω, i.e., don't simply report the normalized values). Label your Smith chart clearly, in particular convincing me you used a calculator as little as possible.