The Cooper Union
Department of Electrical Engineering
ECE135 Engineering Electromagnetics
Exam I
March 8, 2012

Time: 2 hours. **Closed book, closed notes. Calculator provided.**

Part I: Maxwell's Equations and Constitutive Relations. 20pts. total.

1. State Maxwell's equations in **differential** form for the following cases:
   (a) In the time-domain, in the most general case.
   (b) In the phasor domain, assuming a linear homogenous isotropic lossy medium (σ, μ, ϵ), but $\rho = 0$.
   (c) In the frequency-wavenumber domain for a linear homogeneous isotropic lossless medium (μ, ϵ) with $\rho = 0$, $\vec{J} = \vec{0}$.
2. Write the **differential** formula that defines steady current. In classical circuit theory, this yields what famous rule?
3. For the case of static fields, write Ampere's Law in **integral** form.
4. Write the general relation between $\vec{D}$, $\vec{E}$, and $\vec{P}$. Give the MKSA units and the **name** for **all** quantities (including symbols **other** than $\vec{D}, \vec{E}, \vec{P}$) that appear in the formula.
5. Write the general relation between $\vec{B}$, $\vec{H}$ and $\vec{M}$. Give the MKSA units and the **name** for **all** quantities (including symbols **other** than $\vec{B}, \vec{H}, \vec{M}$) that appear in the formula.

Part II: Answer all problems.

1. **[20 pts.]** The phasor electric field in free-space is given by:

$$\vec{E}\left(\vec{r}\right) = E_0 \sin\left(\frac{\pi z}{5}\right) \exp\left(-j\frac{2\pi}{\sqrt{5}}y\right) \hat{a}_x \quad [V/m]$$

   (a) Compute the frequency in Hertz. Name the equation you are using. *Hint:* Related to the wave equation, but not exactly the wave equation.

   (b) In the remainder of the problem, you can write you answer in terms of ω, ϵ_0 and μ_0, as necessary, instead of plugging in values in a calculator. Find the magnetic field, $\vec{H}\left(\vec{r}\right)$.

   (c) Show that perfect conductors may be placed at the $z = 0$ and $z = 5m$ planes, without effecting the field in the region between, but not at some other position z_0 in the range $0 < z_0 < 5m$.

   (d) Find the surface charge and current densities at $z = 0$ and $z = 5$ planes.

   (e) Can we insert a perfect conductor at any $x = x_0$ or $y = y_0$ positions without perturbing the fields? Be specific- either justify why we can, and where we can, or clearly indicate ONE condition (you don't have to list all, any one would suffice) that would be violated if we did.

   (f) Compute the time-average Poynting vector. Identify the direction of (real) power flow.

2. **[10 pts.]** Refer to Figure 2: a conducting cylinder of radius R is partially embedded in a magnetic material (μ), partially exposed to air. A constant total current I flows on the surface of the conductor, along the $+z$-axis.

   (a) Determine $\vec{B}$ and $\vec{H}$ in the region $r > a$.

   (b) You do not have to actually compute $\vec{J}_s$, **but** answer the following about it: is it the same or different on both sides (i.e., the part adjacent to μ and the part adjacent to air)?

3. **[10 pts.]** The electric field in free-space due to a dipole (point charges $\pm Q$ at locations $\pm\frac{d}{2}\hat{a}_z$, with $p = Qd$) is **approximated** for large $|\vec{r}|$ by (spherical coordinates):

$$\vec{E}\left(\vec{r}\right) = \frac{p}{4\pi\epsilon_0 r^3}\left(\hat{a}_r 2\cos\theta + \hat{a}_\theta \sin\theta\right)$$

   (a) The exact charge distribution should be 0, except at $\pm\frac{d}{2}\hat{a}_z$. Compute the charge distribution $\rho\left(\vec{r}\right)$ from the approximate $\vec{E}\left(\vec{r}\right)$ (for $r \neq 0$ of course) and check if (where) it is 0.

   (b) Check whether or not the approximation $\vec{E}\left(\vec{r}\right)$ is itself a valid field, i.e., it satisfies what an electrostatic field needs to satisfy.

   (c) Compute the total charge in a sphere of radius R, directly from $\vec{E}$ (i.e., not from your answer to part **a** above). If not 0 (the exact answer), then answer should at least be constant as a function of radius R. Check that.

4. **[15 pts.]** Refer to Figure 4. Segments of two transmission lines are connected to a resistor R_L at one end, and to a battery, switch and 200Ω resistor at the other end. The switch closes at $t = 0$, and opens at $t = 5ns$. The voltage measured across the 200Ω resistor, $v(t)$, is shown. The first transmission line segment has characteristic impedance $Z_1 = 200\Omega$, and the second has unknown impedance Z_2. Each line has wave velocity $v = 1 \times 10^8 m/s$. The lengths are unknown.

   (a) Find the lengths ℓ_1, ℓ_2 of the transmission line segments.

   (b) Find the characteristic impedance Z_2 of the second line.

   (c) Find the load resistance, R_L.

   (d) Find the voltage and time of arrive of the next pulse to be measured across the 200Ω resistor.

   (e) The method employed here is called (3 words): _______.

5. **[5 pts.]** The propagation constant of a lossy transmission line at a fixed frequency is $\gamma = 3 + j4\ (m^{-1})$. Express the attenuation constant in dB/m, and the wavelength in m.

6. **[20 pts.]** A 100Ω lossless transmission line is terminated in an unknown load impedance Z_L. The location of the minimum voltage nearest to the load occurs at $0.4m$ from the load, with a value of $2V$, and the first maximum occurs at $1.0m$ from the load, with a value of $10V$.

   (a) Sketch the standing wave diagram, up to a distance of $3m$ from the load.

   (b) Specify the wavelength.

   (c) What is the location of the current minimum nearest to the load?

   (d) Compute the VSWR, in dB.

   (e) Find Γ_L, the reflection coefficient at the load (either polar [radian, not degree] or rectangular form).

   (f) Find the load impedance Z_L (must be in rectangular form).

   (g) Find the effective impedance seen at the voltage maximum. *Note:* You do not need the impedance transformation formula to do this.