Emergence of Cooperation using Commitments and Complex Network Dynamics

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***Abstract*—In this paper, our goal is to achieve the emergence of cooperation in self-interested agent societies operating on highly connected scale-free networks. The novelty of this work is that agents are able to control topological features during the network formation phase. We propose a commitment-based dynamic coalition formation approach that result in a single coalition where agents mutually cooperate. Agents play an iterated Prisoner's Dilemma game with their immediate neighbors and offer commitments to their wealthiest neighbors in order to form coalitions. A commitment proposal, that includes a high breaching penalty, incentivizes opponent agents to form coalitions within which they mutually cooperate and thereby increase their payoff. We have analytically determined, and experimentally substantiated, how the value of the penalty should be set with respect to the minimum node degree and the payoff values such that convergence into optimal coalitions is possible. Using a computational model, we determine an appropriate partner selection strategy for the agents that results in a network facilitating the convergence into a single coalition and thereby maximizing average expected payoff.**

***Index Terms*—Scale-free network, emergence, complex network dynamics, multiagent coalition, commitment.**

I. Introduction

There has been a great deal of interest in the multiagent systems (MAS) community about the emergence and maintenance of cooperation among artificial agents [1], [2]. One of the challenging questions addressed in these works is to design autonomous systems in which agents work together to achieve common shared goals. For example, in an emergency disaster management scenario cooperation among the agents in the MAS is required to perform joint tasks [3]. The heterogeneous agents in this scenario are driven by their own local goals. Therefore, it is important to establish cooperative behavior with regard to a global goal for maximizing reward.

Traditionally the tension between personal and social goal is modeled by the Prisoner's Dilemma (PD) game in which the only dominant strategy equilibrium is defection which is not pareto efficient [4]. The PD game offers a powerful metaphor for understanding the challenges of the emergence of cooperation in the face of myopic selfish behavior. In PD, selfish and rational agents try to maximize their utility while interacting with each other. When the PD game is played repeatedly among two agents, it has been shown that the "tit-for-tat" strategy facilitates cooperation [5]. However, in MAS, repeated interaction does not guarantee the evolution of cooperation [6]. Moreover, high connectivity among the nodes results in less cooperation [7].

In this paper, our primary goal is to facilitate the emergence of cooperation in large MAS operating on scale-free (SF) networks where such cooperation helps maximize the global utility of the MAS. We consider the agents in the SF networks have high connectivity and play an iterated PD game with their immediate neighbors. To achieve this goal we propose **a commitment-based dynamic coalition formation approach that leverages the complex network dynamics**.

Dynamic Coalition Formation: Coalition formation provides a mechanism for promoting cooperation in complex networks [8], [2]. A coalition is defined as a group of agents who have decided to cooperate in order to perform a common task. By increasing the organizational level through coalitions, cooperation can be enhanced and maintained. Our primary contribution in this paper is a dynamic coalition formation approach that is based on commitment between agents. A commitment is a promise that an agent offers to another agent in order to influence that agent's strategy. An agent makes use of commitments to exploit the strength of its own strategic position [9]. It has been shown in [1] that commitment can be used to foster cooperation among self-interested agents in non-iterated PD game. Typically a commitment proposal includes a penalty to ensure that the breach of commitment would result in incurring a cost [1]. We enable the self-interested and rational agents to offer commitments to their wealthy neighbors with whom they intend to form coalitions. The agent that offers a commitment bears the cost of maintaining the coalition and promises to pay a penalty should it decide to leave the coalition. The penalty threshold is set such that it provides sufficient incentive to an otherwise non-cooperative neighbor agent to form coalition and thereby cooperate. An agent moves into a different coalition with better social benefit if it is capable of paying the penalty.

In a networked interaction scenario, the challenge is to determine a penalty that facilitates the convergence into a single coalition and at the same time is high enough to incentivize the opponents to form coalitions. We analytically show how the penalty could be set based on the minimum number of immediate neighbors or minimum node degree of the SF network and the payoffs; and provide a sufficient condition that requires to be fulfilled in order for optimal coalitions to emerge.

Complex Network Dynamics: Our secondary contribution in this work is that we investigate the effect of the complex network dynamics over the commitment-based dynamic coalition formation approach. It has been shown previously that although defection is the dominant strategy in the iterated PD game [6], the likelihood of cooperation is remarkably increased if the agent interaction is constrained by the underlying network topology [10], [11]. However, in these approaches agents neither form the network nor use the network dynamics to enhance the emergence phenomenon. These works start by assuming a pre-established static complex network platform and then employ agents on the nodes of the network for mutual interactions. In our work, instead of assuming a given network we enable the agents *to form the network by choosing their interaction partners*. We determine the topological insights that, when embedded into agent partner selection strategy, result in a network always leading towards the emergence of a stable single coalition. In order to gain the topological insights for network formation, we develop a computational model and study how our dynamic coalition formation algorithm performs on various types of SF networks by varying the minimum node-degree, degree-heterogeneity and the clustering coefficient. Specifically, we investigate how a dynamical process of a network, namely the coalition formation, is influenced by its structural properties.

To summarize, in this paper we emphasize the significance of employing "network thinking" by the agents to control their dynamics and the dynamical processes of the network. **This work advances the state of the art by (i) developing a commitment-based dynamic coalition formation approach, (ii) by providing an analytical study about how an effective commitment mechanism is related to the topology of the network and (ii) by determining the topological insights for the agents to choose their interaction partners to form a dynamically growing SF network that enhances the overall cooperation with maximized average expected utility**.

The remainder of this paper is organized as following. We first discuss the relevant literature in section II followed by a description of the two network models for studying the dynamical properties of the SF network. Then we present our commitment-based coalition formation approach in section III. We provide an extensive computational study in section IV and finally conclude with a summary of our observations and discussion of future work in section V.

II. Related Works

The dynamic coalition formation approach in this paper is suitable for large networks. It differs from the existing coalition/team formation approaches in the MAS research community that require the agents to consider all other agents in the network making the process computationally intractable for large networks [12], [13]. Moreover, their agents are constrained to stay in a coalition until the goals of the coalition are accomplished. While these works emphasize the design of negotiation protocols and efficient task distribution, our goal is to promote cooperation at the network level. Our commitment based approach also differs significantly from the existing research works in this area that address the issue of formalization and implementation of commitment mechanisms in MAS interactions [14].

Our approach is inspired from [1] in which the use of commitment is shown to facilitate the emergence of cooperation in a population of agents that play non-iterated PD game. In [1], a variant of the PD payoff matrix is defined to incorporate the penalty and commitment management cost and thereby to provide sufficient incentives for the agents to consider the advantage of mutual cooperation. Their work is based on an unstructured population with random interactions among the agents that use a social learning model and mutation for strategy adaptation. However, they did not consider the effect of their approach in iterated PD game and the role of network topology.

[2] and [8] use a single coalition emergence approach to achieve full cooperation in complex networks. [2] uses a tax collection and information sharing model that require multi-hop communication with high overhead while [8] uses a centralized voting method to decide the strategy of the coalition members.

A work close to our network formation approach is done by [15] that study the emergence of cooperation using a network growth model based on an evolutionary preferential attachment algorithm. This work provides a useful understanding about how the microscopic dynamics could lead to the coevolution of the structure and the macroscopic behavior of the SF network. However, the emergence of *full cooperation* seems to be impossible if the payoff for the temptation to defect is larger than the payoff for the reward.

A parallel thread of research involves studies by physicists on the issue of cooperative behavior among selfish agents over complex networks in the framework of evolutionary game theory. [10] shows that the growth and preferential attachment rule of the SF network significantly enhance the cooperative behavior. [11] studies the impact of average degree on the outcome of the PD game played over SF, small-world and random networks. The effect of high clustering to enhance cooperation over the SF network has been studied in [16].

The above research, conducted by eclectic disciplines, emphasize the fact that addressing the topological issues of complex networks for enhancing the cooperation is as important as formulating appropriate interaction strategies for the agents.

A. Network Models

In the context of social systems and in many real world applications we observe that the network exhibits both node degree-heterogeneity and high clustering. The standard Barabasi-Albert (BA) SF network model [17], however, suffers from low clustering. Moreover, the heterogeneous degree-distribution of the BA model is fixed by the constant power law scaling-exponent. Hence, to emulate more realistic scenarios, we consider the following two SF network models that we use to build a computational model for studying the performance of our approach and to gain insights about the impact of topological features over the process of coalition emergence:

BA Model

The BA SF model [17] is formed as follows:

(i) Growth: Starting from m_0 nodes, at every time step a new node is added with *m* ($m <= m_0$) edges which connect between the new node and m different previously existing nodes.

(ii) Preferential Attachment: A node *i* is chosen to be connected to the new node according to the probability $\prod_{n \to i} = \frac{A+k_i}{\sum_j (A+k_j)}$ where k_i is the degree of node *i* and **A** is a *tunable parameter* representing the initial attractiveness of each node.

Extended BA Model

The extended model [18] follows the growing process of the BA model that starts with m_0 nodes. At every time step a new node *i* is added to the network and gets connected with *m* ($m <= m_0$) of the previously existent nodes. The first link of node *i* is added to node j of the network (with $j < i$) following the preferential attachment rule of the BA model. The remaining $m-1$ links are added in two different ways: (a) with clustering probability *p* the new node *i* is added to a randomly chosen neighbor of node *j* and (b) with probability $(1-p)$ node *i* gets connected to one of the previously existing node using the preferential attachment rule again. This procedure ensures a degree distribution of $p(k) \sim p^{-\gamma}$ with a tunable clustering coefficient.

III. Commitment Based Coalition Formation

In this section, we present the formal model for our proposed coalition formation approach.

A. Model

The agent interactions in the MAS are specified by an undirected graph $G(V, E)$ where V is the set of vertices (or nodes) and $E \subseteq V$ x V is the set of edges. Each node corresponds to an agent.[1]. The numbers of nodes are referred by n. Once the graph or the network is formed by the agents it becomes fixed. Two nodes v_i and v_j are neighbors if $(v_i, v_j) \in E$. The neighborhood $N(i)$ is the set of nodes adjacent to v_i. That is, $N(i) = \{v_j | (v_i, v_j)\} \in E \subset V$ and $|N(i)|$ is the degree of node v_i.

The graph follows scale-free property in which the distribution of node degree follows a power-law, $N_d \propto d^{-\gamma}$, where N_d is the number of nodes of degree d and γ is a constant. We have described the scale-free graph models in detail in section II.

Our proposed decentralized coalition formation approach requires the agents to communicate only with their immediate neighborhood to form coalitions. We assume that agents are self-interested and rational. To initialize, we enable the agents to form the network by choosing their interaction partners dynamically. The adjacent agents (within single-hop distance) are defined as the *neighbors*. Every agent is equipped to play a *2-person* iterated PD game with each one of its neighbors and their interactions are represented by the network links. The agents start playing the PD game after the network is formed and we consider the final network as a closed system.

Agent i's payoff is denoted by $u(i, j)$ which agent i obtains by playing a PD game with its neighbor j. After every round of the game, the payoff received by playing the PD game with the neighbors gets accumulated and the accumulated payoff is defined as $\sum_{j=1}^{m} u(i, j)$, where j refers to the neighbors of i. We assume that agents know the accumulated payoff of their neighbors. Every agent has a fixed strategy for each one of its neighbors, which is either to cooperate (C) or to defect (D). In a *2-person* PD game setting these two strategies intersect at four possible outcomes represented by designated payoffs: R (reward) and P (punishment) are the payoffs for mutual cooperation and defection, respectively, whereas S (sucker) and T (temptation) are the payoffs for cooperation by one player and defection by the other. The payoff matrix is represented by Table I. For the PD game, the payoffs satisfy the condition $T > R > P > S$ and for iterated PD's we require $T + S < 2R$.

TABLE I
Payoff Matrix for the Prisoner's Dilemma Game

	C	D
C	(R,R)	(S,T)
D	(T,S)	(P,P)

The iterated PD game proceeds in rounds and each round has three phases: (i) the agents play the game with all the neighbors using fixed strategies and compute the accumulated payoff, (ii) based on the payoff information of the neighborhood, the agents form/join coalition and (iii) update the strategies used in the coalition formation algorithm.

We define two types of agents: independent agents and coalition member agents. These two types are mutually exclusive. Initially all the agents are assumed to be independent. $Ind(v_i)$ refers to a set of independent agents $i : v_1, v_2, ..., v_n$. An agent v_i makes a commitment $Comm(v_i, v_j)$ to its neighbor agent v_j and forms a coalition $Coa(v_i, v_j)$. A coalition member agent is committed to the coalition; it always cooperates with its neighbors belonging to the same coalition and defects with others. In other words, we implement the strategy: *no cooperation without commitment.* However, an independent agent takes the interaction strategy that the majority of its neighbors adopted in the previous round. We define the coalition formation process and the algorithm in the next sub-section.

B. Definitions, Algorithm and Theorem

Definition 1. *Commitment: An agent v_i makes a commitment $Comm(v_i, v_j)$ to its largest accumulated payoff neighbor v_i*

[1]Throughout the paper, we use agent and node interchangeably.

with whom it intends to form a coalition. The commitment proposal includes the following:

1) v_i *would bear a small management cost* β *to maintain the coalition*
2) v_i *would pay a penalty* α *if it unilaterally breaks the coalition and vice versa*

Definition 2. *Coalition Formation: If the accumulated payoff of an independent agent* $v_i \in Ind(v_i)$ *is smaller than the accumulated payoff of its neighbor* v_j *whose payoff is the largest in* v_i*'s neighborhood, i. e., if* $\sum u(v_i) < \sum u(v_j)$ *and* $(v_i, v_j) \in E$*, then* v_i *forms a coalition* $Coa(v_i, v_j)$ *with* v_j *by making a commitment* $Comm(v_i, v_j)$ *as defined in Definition 1. Agent* v_i *cooperates with the members of the same coalition and defects with others belonging to it's neighborhood.*

As in [1], the management cost is very small compared to the reward, i. e., $\beta << R$ and the penalty α is larger than the temptation payoff, i. e., $\alpha > T$ in order to offer enough incentive to an opponent to form a coalition.

Initially there would be multiple coalitions where agents may find it profitable to leave their existing coalitions and join new ones. We define the inter-coalition dynamics as following:

Definition 3. *Inter-Coalition Dynamics: If the accumulated payoff of a coalition agent* v_i *is smaller than the accumulated payoff of its neighbor* v_j *that belongs to another coalition, whose payoff is the largest in* v_i*'s neighborhood, i. e., if* $\sum u(v_i) < \sum u(v_j)$*,* $(v_i, v_j) \in E$ *and* $Coa(v_i) \neq Coa(v_j)$*, then* v_i *leaves its existing coalition and joins the coalition of* v_j *if the following condition is fulfilled:*

$$\frac{\sum u(v_i)}{2} > \alpha$$

Definition 4. *Coalition Convergence: After repeating the* ***Inter-Coalition Dynamics*** *phase multiple times the network converges into a single coalition where no agent either finds it beneficial to leave the existing coalition or to form a new one.*

Procedure 1: NetworkFormation

Require: m_0 initial nodes
Require: number of edges (m) of the newly connected node: $m \leq m_0$
1. setInitialAttractiveness() = A;
2. setClusteringProbability() = p;
3. implementBAModel;
4. WHILE($m \leq m_0$)
5. {linkToNode(i): $\prod_{n \to i} = \frac{A+degree_i}{\sum_j (A+degree_j)}$;}
6. end;
7 implementExtendedBAModel;
8. linkToNode(i): $\prod_{n \to i} = \frac{A+degree_i}{\sum_j (A+degree_j)}$;
9. WHILE($m \leq m_0 - 1$)
10. {linkToNeighborOfNode(i)withProbability(p);
11. linkToNode(i)withProbability(p-1):
 $\prod_{n \to i} = \frac{A+degree_i}{\sum_j (A+degree_j)}$;}
12. end;

Procedure 2: InitialCoalitionFormation

Require: Accumulated payoff is transparent only to immediate neighbors
Require: All the agents are Independent
1. networkFormation();
2. randomStrategySelection();
3. playPDGamewithNeighbors();
4. **FOR** each agent i:= 1 to n
5. **IF** maximumPayoffNeighbor(j) AND
6. payoff(i) < payoff(j)
7. offerCommitmentTo(j);
8. formCoalitionWith(j);
9. payManagementCostBy(i);
10. **ELSE**
11. remainIndependentAgent(i)
12. **END FOR**

Algorithms: Our Coalition Formation with Network Dynamics Algorithm (CFNDA) has 3 steps: network formation, initial coalition formation and decentralized coalition formation described below by procedures 1, 2 and 3 respectively.

Network Formation: In the beginning agents choose their interaction partners and form the network as described in Procedure 1. Agents are enabled to set the values of their initial attractiveness parameter (A) and the clustering probability (p). Agents may either form the network according to the BA model (lines 3-6) or may use the extended BA model (lines 7-12). In the BA model, all the links (*m*) of the new node are connected to the existing nodes using the preferential attachment rule (line 5). On the other hand, in the extended BA model only the first link of the new node is added using the preferential attachment rule (line 8). The remaining links of the new node ($m_0 - 1$) are added to the randomly chosen neighbors of the first neighbor of the new node with the probability *p* (line 10) or using the preferential attachment rule with the probability *p-1* (line 11). By varying the value of A, the degree-heterogeneity of the resultant network can be controlled and *p* determines the clustering level of the extended BA model. Using a computational model described in section IV, we determine how the agents should set these two topological parameters such that the resultant network enhances the emergence of a single coalition when agents form coalitions using algorithms 2 and 3.

Initial Coalition Formation: Procedure 2 depicts how initial coalitions are formed at the beginning of the game. Every agent starts out as an independent agent and there is no coalition. Agents choose their interaction strategy randomly and generate the payoff according to the payoff matrix in Table I by playing a *2-person* PD game with each one of its neighbors (lines 2-3). Then in lines 5-9, for every agent if the largest payoff neighbor j's accumulated payoff is larger than the agent i's payoff, it offers commitment to j and forms a coalition. It also bears the management cost of that coalition. An agent without any coalition members remains independent (line 11). After the first round, there would be

Procedure 3: Decentralized Coalition Formation Algorithm

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Require: Accumulated payoff is transparent only
to immediate neighbors
1.  initialCoalitionFormation();
2.  playPDGamewithNeighbors();
3.  FOR each agent i:= 1 to n
4.    IF coalitionAgent(i) AND
5.    maximumPayoffNeighbor(j) AND
6.    payoff(i) < payoff(j)
7.      IF (notIndependentAgent(j))
8.        IF u(i)/2 > α
9.        {offerCommitmentTo(j);
10.       joinCoalitionOf(j);
11.       payManagementCostBy(i)};
12.   ELSE IF (independentAgent(j))
13.     GOTO lines 9-11;
14.   IF (coalitionAgent(i) AND
15.   disconnectedFromCoalition(i))
16.     {becomeIndependentAgent(i)};
17.   IF (independentAgent(i))
18.     GOTO lines 5-13;
19.   mutation();
20. END FOR
```

multiple coalitions. The number of coalitions will depend on the size of the network.

Decentralized Coalition Formation: At the beginning of every round each agent plays the PD game and employs the coalition strategies to join/leave/switch or form a coalition according to Procedure 3. In lines 4-11 every coalition member agent i joins the coalition of it's largest payoff neighbor j if one-half of i's payoff is larger than the penalty. Agent i offers a commitment to j and bears the management cost of the coalition. If j is an independent agent, then i forms a coalition with it by offering a commitment and bearing the management cost (lines 12-13). If agent i is a coalition member agent but is disconnected from its coalition members (when an agent does not have any one-hop link to other members of its coalition, then it is considered to be disconnected from its coalition), it becomes an independent agent (lines 15-16). However, if i is an independent agent then it forms a new coalition according to lines 5-13.

Mutation: It is possible that some agents might become stable within sub-optimal coalitions where the majority of the neighbors do not belong to the agent's coalition. In order to allow these agents to move to optimal coalitions (which maximizes their payoff), they are enabled to explore the strategy space with a small probability. If the majority of a coalition-agent's neighbors are not its coalition members, that agent becomes independent if one-half of its payoff is larger than the penalty.

Proposition 1. *For any connected graph G with n nodes and (sufficiently) high penalty ($\alpha >$ temptation payoff), the agents increase their payoff through the coalition formation process.*

Proof:

Let us consider three agents a_1, a_2 and a_3 are playing an iterated PD game with their immediate neighbors. Both a_2 and a_3 are the neighbors of a_1. Assume that after the first round of the game, the accumulated payoff of a_2 is the largest in a_1's neighborhood, i. e., $\sum u(a_2) > \sum u(a_3) > \sum u(a_1)$. Now according to the CFNDA, agent a_1 will form a coalition with a_2 by making a commitment and will start cooperating with the same coalition members in its neighborhood. This mutual cooperation may increase a_1's payoff. However, it is possible that a_1 was a defector with its other neighbors; in that case its payoff would not increase after joining a_2's coalition.

Now, in the next round of the game, after joining the coalition if the majority of a_1's neighbors belong to the same coalition, its payoff further increases through mutual cooperation. With this increased payoff a_1 will eventually attract its non-coalition neighbors to join a_1's coalition. This would result in a maximum payoff of a_1. On the other hand, if the majority of a_1's neighbor do not belong to its coalition and if one of the neighbors' payoff happens to be larger than that of a_1's payoff, then a_1 will leave its existing coalition and will form/join that neighbor's coalition if $\sum u(a_1)/2 > \alpha$ condition is satisfied. In the new coalition, a_1's payoff is expected to increase further because its coalition partner is the wealthiest in a_1's neighborhood and thereby it would attract more agents to join its coalition increasing the likelihood of mutual cooperation. However, this process may lead a_1 to a situation where it may get stuck in a sub-optimal coalition. The mutation strategy of the CFNDA could resolve this problem by allowing a_1 to move towards more beneficial coalitions and thereby increase its payoff.

■

Using Proposition 1 we now prove that the coalition formation algorithm guarantees maximum average expected payoff in any scale-free random graph.

Theorem 1. *For any random scale-free graph G with n nodes and (sufficiently) high penalty ($\alpha >$ temptation payoff), the coalition formation process converges into a single coalition and maximizes the average expected payoff, if the minimum node-degree (min_d), penalty (α), reward (R) and punishment (P) payoffs fulfill the following condition:*

$$min_d \geq \frac{4\alpha}{R+P}$$

Proof:

According to Proposition 1, it is sufficient to prove that in scale-free random graphs either a node has earned the maximum payoff (when all of its neighbors belong to the same coalition) or one-half of its payoff is larger than the penalty to move to another coalition leading towards the convergence into a single coalition that maximizes its payoff.

In any random scale-free graph, there are few high-degree nodes linked by many low-degree neighbors. Since initially the nodes interact based on randomly assigned strategies, the interaction partners of any node would be a uniform mixture of cooperators and defectors. This leads the high-degree nodes to generate high accumulated payoffs in their neighborhood. Therefore, most or all of the neighbors of the high-degree nodes form coalitions with them resulting all (or almost all)-cooperation sub-graphs around the high-degree nodes.

Let us assume that a_2 and a_3 are two high-degree nodes in a_1's neighborhood and that initially in the first round of the game a_1 has formed a coalition with a_2. Also assume that the degree of a_3 is larger than the degree of a_2, i.e., $d(a_3) > d(a_2)$. Therefore the number of cooperating coalition members of a_3 should be larger than that of a_2. This would increase the payoff of a_3. Hence, in the next round of the game a_1 finds it profitable to leave the coalition of a_2 and join the coalition of a_3 and thereby increase its payoff if $\sum u(a_1)/2 > \alpha$. Now we will prove that this condition is always satisfied until the entire network converges into a single coalition where no node has any motivation to leave the coalition.

Let us assume that in the first round of the game when a_1 belonged to a_2's coalition, x number of neighbors of a_1 cooperates with it (including the node a_2) and the remaining nodes of its neighborhood (which is at least, minimum degree of a_1 or $min_d(a_1)-x$) belong to different coalitions and hence are defectors. Therefore, after the end of the first round, the accumulated payoff of a_1 would be

$$\sum u(a_1) = x * R + (min_d(a_1) - x) * P$$

Now, in the next round, for a_1 to move to a_3's coalition for maximizing it's payoff, a_1 needs to satisfy the following condition

$$\frac{x * R + (min_d(a_1) - x) * P}{2} > \alpha$$

We know that $\alpha > R > P$, hence for the above condition to be satisfied, both x and $min_d(a_1)$ have to be sufficiently large. Since initially there were equal number of cooperators and defectors, it is expected that at least half of a_1's neighbors would belong to the same coalition. Therefore,

$$\frac{\frac{min_d(a_1)}{2} * R + (min_d(a_1) - \frac{min_d(a_1)}{2}) * P}{2} \geq \alpha$$

$\implies \frac{min_d(a_1)}{4} * (R + P) \geq \alpha$
$\implies min_d(a_1) \geq \frac{4\alpha}{R+P}$

Therefore, if the minimum node-degree of G fulfills the above condition, the agents would tend towards beneficial coalitions, thereby increasing the number of cooperators in their neighborhood, until all the agents converge into a single coalition in which mutual cooperation guarantees the maximization of the average expected payoff of the agents. ∎

IV. Computational Model and Results Analysis

Our goal here is threefold: (a) computationally validate our approach by showing that if the penalty is set according to the condition provided in Theorem 1, convergence into optimal coalitions is possible, (b) show that the performance of our approach for the emergence of cooperation in highly connected SF networks is better than two state-of-the-art approaches and (c) determine the topological insights that agents could use to choose their partners such that the resulting network facilitates cooperation.

For comparison, specifically we use two state-of-the-art action update rules, namely the *imitate-best-neighbor (IB)* [4] and the *stochastic imitate-random-neighbor (SA)* [19], that has been shown to facilitate the evolution of cooperation on SF networks [6]. We study the performance of these rules over varying-degree SF networks. Although these two approaches do not use coalition formation for the evolution of cooperation, we study these to underscore the challenge of achieving cooperation in highly connected networks. We use a computational model to conduct extensive simulations for our coalition formation approach by varying the node degree-heterogeneity and the clustering coefficient of the BA and the extended BA model. We increase the value of the initial attractiveness parameter (A) in order to vary the degree-heterogeneity of the network and increase the value of the clustering probability *p* (used in the extended BA model) to generate medium ($p = 0.5$) and high-clustering ($p = 0.1$) networks respectively; and observe the state of convergence of the coalitions. Also we investigate how the average expected payoff increases in each type of network instantiation. Heterogeneity is measured by the standard deviation of the degree distribution.

A. Simulation Setup

Our network consists of 5000 agents represented as nodes in the SF network. A link between two nodes of the network indicates that the agents interact and play the PD game. We set the default minimum node degree as 10 in both models ($m = 10$).

We assign the following values for the payoffs: T = 5, R = 3, P = 1 and S = 0. According to Theorem 1, the value of the penalty (α) is set to 10. We choose the value of the management cost (β) as 0.005.

All the results reported are averages over 100 realizations for each network for different values of the network parameters (e.g., degree-heterogeneity, clustering coefficient etc.). Each simulation consists of 500 time steps where a time step refers to a single run of the program. The mutation rate is set to 0.05.

TABLE II
IB & SA Rules: The average no. of cooperators (#Coop) and average expected payoff (ExPoff) over 100 realizations of the network for various values of the minimum node degree. Both p and A are zero.

	IB		SA	
Min-Degree	#Coop	ExPoff	#Coop	ExPoff
1	1337.1	-75.49	1238.28	-78.37
2	2824.55	-31.43	2304.17	-47.41
3	400.81	-111.80	506.15	-108.04
5	0.0	-124.77	49.9	-123.53
10	0.0	-125.31	0.0	-124.92

B. Simulation Results

Evolution of Cooperation vs. Minimum Node Degree: We study the effect of IB and SA action update rules over the final fraction of cooperators by varying the minimum node degree of the network. We use the BA model ($p = 0.0$) with the initial attractiveness parameter A set to 0. Table II shows that the evolution of cooperation occurs only when the network is sparse. The average number of cooperators drops to zero for both update rules when the minimum node degree exceeds 5. This also results in very low average expected payoff of the network. However, from Table III we observe that our proposed commitment-based dynamic coalition formation approach is able to increase mutual cooperation by converging into a single coalition and to maximize the average expected payoff of the agents in highly connected networks (when minimum node degree is 10).

Convergence of the Network: To observe the performance of our coalition formation approach over a low-clustering SF network, we set the initial attractiveness parameter A = 0. Then we gradually increase the value of A from 0 to 10,000. From Table III we observe that as A increases, the degree-heterogeneity of the network decreases and the likelihood of convergence into a single coalition increases. Therefore, clearly the agents should benefit by selecting their partners by setting a large value of A during the network formation phase (Procedure I in Section III). It guarantees the convergence into a single coalition and maximizes the average expected payoff. The average global clustering coefficient of the network is very small, as expected from the standard BA network.

We use the extended BA model to investigate the effect of high clustering and degree-heterogeneity over the emergence of a single coalition. For both medium-clustering (p = 0.5) and high-clustering (p = 1.0) networks we increase the initial attractiveness parameter A from 0 to 10,000. From Table III we notice that in the medium-clustering network, very large value of A (> 1000) does not necessarily improve the convergence. We get the best convergence when A = 500. Hence our expectations about improving the convergence by controlling the value of A are partially met by the results. On the other hand, in the high-clustering model, very large value of A (> 5000) is required to improve the convergence. In the high-clustering model we see that even a very large value of A does not decrease the degree-heterogeneity of the network significantly. The reason is that according to the extended model of the BA network with p = 1.0, only the selection of the first neighbor can be controlled by the parameter A.

Average Expected Payoff: Figure 1 shows how the average expected payoff varies over the iterations for 3 network types: low-clustering (p = 0.0), medium-clustering (p = 0.5) and high-clustering (p = 1.0). We set the value of A to 0 in these models. We notice that in all these three types of networks the average expected payoff increases and becomes stable at an optimum value. We also notice that irrespective of the network clustering our commitment-based dynamic coalition formation approach improves the average expected payoff of the network and increases it in a similar fashion.

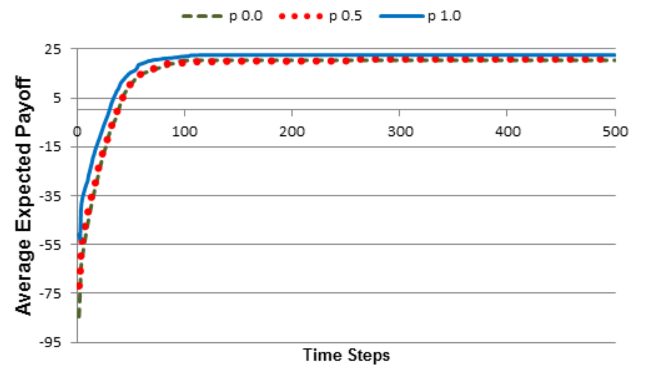

Fig. 1. BA & Extended BA Model: Increase of average expected payoff for various values of the clustering probability p

Discussion: The performance of the commitment-based approach depends on how the value of the penalty is set. The challenge is to determine an appropriate range of the penalty that is high enough to incentivize an opponent (larger than the temptation payoff) but not very high to obstruct the process of convergence into a single coalition. According to the required and sufficient condition in Theorem 1, for a highly connected network with minimum node degree 10 (average node degree 20), this value should be no greater than 10. Using this value we are able to guarantee the convergence. We experimented with different values of the node degree and the penalty (the result is not reported here) and obtained similar result. Therefore, unlike the existing approaches (IB and SA), our approach is able to evolve cooperation and maximize the average expected payoff both in sparser and highly connected networks.

We are able to fine-tune the performance of our approach by enabling the agents to control the topological features (such as degree-heterogeneity and clustering) during the network formation phase. The agents in a MAS choose their interaction partners according to the preferential attachment rule of the BA model. In the extended BA model, a fraction of the nodes (depending on the clustering probability p) form the links according to this rule. In order to guarantee optimal convergence of our approach, agents during the network formation phase need to choose their interaction partners using a large value of the parameter A. This decreases the node degree-heterogeneity and thereby reduces the number of large multiple coalitions that otherwise may lead towards converging into sub-optimal coalitions.

We notice that the average expected payoff in the highly-clustered network (p = 1.0) is not as high as in both the low and medium clustered networks. In highly-clustered network, a node is connected to its neighbor's neighbors. Therefore, the shared management costs across every agent's neighborhood are relatively high compared to low-clustering networks. This indicates that agent's partner selection strategy and the network formation affect the overall social benefit.

TABLE III

BA & Extended BA Model: The average no. of coalitions (#Coa), average expected payoff (ExPoff), average Global Clustering Coefficient (GCC) and average Degree-Heterogeneity (DH) over 100 realizations of the network for various values of p and A.

	BA Model				Extended BA Model							
	p = 0.0				p = 0.5				p = 1.0			
A	#Coa	ExPoff	GCC	DH	#Coa	ExPoff	GCC	DH	#Coa	ExPoff	GCC	DH
0	1.44	20.21	0.02	52.58	1.51	20.74	0.13	52.25	1.96	16.46	0.31	52.35
50	1.18	21.76	0.01	27.54	2.93	19.60	0.12	34.25	3.66	14.64	0.41	50.65
100	1.14	21.74	0.01	25.57	1.08	21.73	0.12	31.43	1.87	20.83	0.42	52.07
500	1.01	21.98	0.01	23.96	1.01	22.08	0.12	29.54	3.61	15.85	0.45	49.21
1000	1.04	21.80	0.01	23.73	1.10	21.56	0.11	29.19	2.99	17.86	0.45	49.42
2000	1.03	21.93	0.01	23.58	1.15	20.62	0.12	29.14	3.11	15.96	0.44	50.07
5000	1.01	21.89	0.01	23.50	1.25	20.28	0.12	28.83	1.89	20.38	0.45	50.23
10000	1.00	21.97	0.01	23.48	1.19	21.09	0.11	28.99	1.93	20.45	0.46	50.53

V. Conclusions and Future Work

In this paper, we describe a commitment-based dynamic coalition formation approach to establish mutual cooperation in a large MAS operating on SF networks. We capture the interactions of the self-interested agents with their immediate neighbors using an iterated PD game. Unlike many previous works that assume given pre-established networks, our agents dynamically choose their interaction partners to form the network. Agents offer a commitment to their wealthiest neighbors in order to form coalitions. A commitment proposal, that includes a high penalty for breaching the commitment, incentivizes opponent agents to form coalitions inside which they mutually cooperate and thereby increase their payoff. Our main findings are as follows:

- We have analytically determined, and experimentally substantiated, how the value of the penalty should be set with respect to the minimum node degree and the payoff values such that convergence into optimal coalitions is possible.
- Our approach is a contribution to the state-of-the-art as it is able to evolve cooperation in highly connected networks.
- Also our work is novel in that our agents are capable of controlling some topological features of the network that results in better convergence and increased average expected payoff.

In future we plan to extend our model for incomplete information games where agents do not know their neighbors payoff.

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