Discrete-Time

\[ n \in \mathbb{Z}, \omega \in [-\pi, \pi) \]

Convolution: \( y = h * x \):

\[ y[n] = \sum_{k=-\infty}^{\infty} h[k] x[n-k] \]

If \( h \) has length \( M \) and \( x \) has length \( N \) then \( y = h * x \) has length \( N + M - 1 \)

\[ z \text{-transform:} \]

\[ X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} \]

DTFT (Discrete-Time Fourier Transform) and IDTFT:

\[ X(\omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} \]

\[ x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega) e^{j\omega n} d\omega \]

Other results and properties:

- Definition of \( \delta[n] \) and \( u[n] \)
- Convolution theorem: \( y = h * x \) has z-transform \( Y(z) = H(z)X(z) \) and DTFT \( Y(\omega) = H(\omega)X(\omega) \)
- z-transform pairs: \( \delta[n] \leftrightarrow 1, u[n] \leftrightarrow \frac{z}{z-1}, \alpha^n u[n] \leftrightarrow \frac{z}{z-\alpha} \)
- z-transform and DTFT related via \( z = e^{j\omega} \)
- BIBO stability iff all poles inside unit circle (technically, this presumes causal systems exclusively)

Filters:

- FIR filters: definition; transversal filter structure and difference equation corresponding to \( H(z) = h_0 + h_1 z^{-1} + \cdots + h_N z^{-N} \); \( N \) = order and \( N + 1 \) = length
- IIR filters: definition; direct form II and direct form II transposed structures and difference equation (time-domain expression, not the circuit diagram) corresponding to rational function \( H(z) \); \( N \) = order = \# delays in the direct form II structure
For case of discrete periodic time: $0 \leq n \leq N-1$, and frequency indices $0 \leq k \leq N-1$, apply DFT (Discrete Fourier Transform) and IDTFT:

$$X[k] = \sum_{n=0}^{N-1} x[n] \exp\left(-j\frac{2\pi}{N} nk\right)$$

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] \exp\left(j\frac{2\pi}{N} nk\right)$$

Basic result of sampling theory: $f =$ analog frequency (Hz); $f_\text{s} =$ sampling rate (Hz); $\omega =$ normalized digital radian frequency (rad):

$$\omega = 2\pi f / f_\text{s}$$

**Continuous-Time**

$t \in \mathbb{R}, \omega \in \mathbb{R}$

Convolution: $y = h \ast x$

$$y(t) = \int_{-\infty}^{\infty} h(\tau) x(t-\tau) d\tau$$

If $h$ has duration $T_h$ and $x$ has duration $T_x$ then $y$ has duration $T_h + T_x$

Laplace transform:

$$X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt$$

CTFT (Continuous-Time Fourier Transform) and ICTFT:

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$$

Other results and properties:

- Definition of $\delta(t)$ and $u(t)$

- Convolution theorem: $y = h \ast x$ has z-transform $Y(s) = H(s) X(s)$ and CTFT $Y(\omega) = H(\omega) X(\omega)$

- Laplace transform pairs: $\delta(t) \longleftrightarrow 1, u(t) \longleftrightarrow \frac{1}{s}, e^{at}u(t) \longleftrightarrow \frac{1}{s-a}$

- Laplace transform and CTFT related via $s = j\omega$

- BIBO stability iff all poles inside LHP (technically, this presumes causal systems exclusively)
LTI Systems

Definition of linear, time-invariant, causal, BIBO stability.

LTI system characterized in time-domain by convolution with impulse response; in frequency domain, characterized by multiplication with frequency response; in transform domain, characterized by multiplication with transfer function.